Analysis of Revenue Sharing in Mobile Application Market using Game Theoretic Model

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Increasing penetration rate of smartphones makes a mobile application market grow drastically and corporations concentrate effort to stimulate the mobile application market based on their own Operation System (OS) platforms. In order to activate the market, OS platform providers entice application developers to their own OS platforms by sharing a fixed rate of their revenue to application developers. In this paper, we have modelled imbalanced profit structure between OS platform providers and application developers through game theory approach. Then, we have analyzed optimal rate of revenue sharing, which application developers and OS platform providers set respectively.

1. Introduction

Users have moved from conventional feature phones to smartphones. By using smartphones, users can download applications to utilize functions that they want. This means that the mobile industry’s paradigm is shifting from voice-centered services to various application-centered ones. That is, a main provider of mobile service has been changing from restricted providers such as manufacturers and telecommunications companies to individual application developers. Under the previous voice-centered paradigm, calling plan, call quality, design, and performance of mobile phones were the factors of competition. However, under the application-centered paradigm, not only performance of the mobile phones but also OS and application markets becomes critical factors. Unlike the feature phone users who can use only provided functions, smartphone users can download desired functions whenever they needed.

According to Gartner, market research agency, smartphones accounted for 16 percent of total mobile-phone sales in the fourth quarter of 2009. It forecasts that smartphones will account for 26 percent in 2011, and 49 percent in 2015; the smartphones are expected to spread rapidly. In addition, the mobile application market, which was worth $ 3.99 billion in 2010, is predicted that it will grow to $ 9.02 billion in 2011, $ 17.0 billion in 2012, and $ 52.3 billion in 2015, which means that the market is expects to grow more than 10 times. The accumulated number of application downloads associated with the application market by 2010 was approximately 1.1 billion. Gartner forecasts that it would exceed 6 billion in 2012, $10.9 billion in 2013, and $29.4 billion in 2015.

Looking through the currently operating application market, there are a number of application markets such as Apple’s ‘App Store’, Google’s ‘Android Market’, Microsoft’s ‘Window Marketplace for Mobile Market’ and Nokia’s ‘Ovi Store.’

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However, there are only two markets that have constructed a meaningful level of application market; Android market and App Store has secured approximately 730 thousand and 620 thousand applications for each.

Apple’s ‘App Store’, an originator for the application market, is recognized as the most successful model and is becoming the benchmark for other application market. In July 2008, Apple opened its app stores with the launch of an app store. After launching it, Apple got 100 million application download within 2 months and 200 million application download within 3 months. This could happen because Apple had achieved a positive feedback loop. Apple had naturally induced loyal iPod and iTunes customers to the App Store, increasing users in the application market. This made a number of application developers enter into the App Store and they had made many applications, which raised the number of users. In the App Store, application developers utilize apple’s SDK (Software Development Kit) to develop applications which are screened by Apple’s own policy. The features appear in this process are that Apple takes a fully closed strategy for its OS platform, thus the App Store can be viewed as the closed OS platform.

Google’s ‘Android Market’ was the first market that can compete with App Store. The android market opened in October 2008 and began selling applications in February 2009. Google’s Android OS was first introduced in November 2007 with 33 companies, including Google, Qualcomm, Motorola, LG Electronics, Samsung Electronics, etc. The companies had jointly assembled the Android OS with OHA (Open Mobile Alliance). The Android OS is an operating system has been developed and managed by OHA. Each manufacturer and developer can modify the source code of the Android OS to optimize its own products so that it can reduce the costs and durations to develop applications. Also, Android OS can share various contents, including applications. Due to these advantages, Android OS has spread rapidly, and Android Market has been activated fast as well. Main feature of the Android market is that application developers can modify the source code of the Android OS freely, which makes Android market considered as Open OS platform.

As we saw above, rapidly growing application markets tend to build OS platform markets based on their own OS platforms. That is, they use the strong point to attract developers and consumers. This is because the most important aspect of winning the competition is the activation of the application market. Especially, it is important to increase the utilization of applications by gaining more applications compared to other competing application markets. Therefore, OS platform providers are luring application developers to their market segments by allowing application developers to receive a fixed rate of profit from selling applications. Gans (2012) conducted an analysis of the application price by optimizing application developers’ application price problem in case the developers sell their applications directly or via mobile platform to the consumers. In particular, the developers were considered under the constraints which can be caused in case of providing applications through mobile platform.

Given the importance of the application market, however, research on the pricing and revenue sharing issue of the application market has recently been done. Li and Lin (2009), Li (2010), etc. covered price issues similar to those of application price. Li and Lin (2009) analyzed DRM protection policy and price issues of digital contents considering the quality of contents and network environment, using the game theory model. Li (2010) conducted an analysis about optimal price of digital contents related to Web and P2P channels through game theory model. Liu and Tang (2006) analyzed
an optimal revenue sharing rate in mobile value-added service. He and Walrand (2005) established a revenue sharing strategy of internet service provider through game theory.

However, research on the pricing and revenue sharing issues of traditional application market shows that despite the close relevance between application price problems and the issue of revenue sharing, it seems to be hard to consider both of them at the same time. Accordingly, the objective of the present study is to analyze the optimal price of application and revenue sharing rate of OS platform providers. It is followed by modelling a situation where OS platform provider and application developers are under imbalanced profit structure, through game theory. To do so, we apply Stackelberg Game in which application developers determine the price of application first considering the revenue sharing rate of OS platform providers. Then, OS platform providers choose their optimal revenue sharing rate.

The remainder of the paper is organized as follows: Section 2 describes our problematic situation in which we explains the game model. In section 3, we derive the optimal price of applications and the optimal revenue sharing rate of OS platform provider. The general conclusions and suggestions for further research are briefly presented in section 4.

2. Model Framework

2.1 Problem Description

Suppose that application developers use only two types of OS platforms, A and B OS platforms. Basically, each OS platform provider determines a revenue sharing rate on the applications being sold on one’s OS platform market. The two platform providers induce application developers to release applications in their markets by sharing their revenues. That is, each type of OS platform provider decides a rate of revenue sharing to maximize its profits respectively by making application developers release their applications in their OS platform markets. This means that two types of OS platform markets are in competition. In this paper, however, one of two OS platform providers, B OS platform provider, has a fixed rate of revenue sharing. Only the other OS platform provider, A OS platform provider, can choose its optimal revenue sharing rate.

Application developers first decide one of two OS platforms in which they will release applications and then they determine the price of the application on the OS platform market they selected. Application developers actually can sell their applications on both types of OS platform markets. In this paper, however, we suppose that application developers can choose only one OS platform market. Also, it is assumed that application development costs of two types of OS platform are different and the costs of developing applications are equally consistent with uniform distribution. $C_{i,j}$ is developer j’s application development cost on i type of OS platform market and $C_{max}$ is maximum application development cost on i type of OS platform market.

$$C_f^A = U[0, C_{max}^A]$$
\[
f_{c_{j}^{A}}(c_{j}^{A}) = f(c_{j}^{A}) = \begin{cases} 
\frac{1}{c_{\text{max}}^{A}} & 0 < c_{j}^{A} < c_{\text{max}}^{A} \\
0 & o/w 
\end{cases}
\]

(1)

\[
c_{j}^{B} = U[0, c_{\text{max}}^{B}] 
\]

(2)

This study assumes that the maximum cost of application development on A type of OS platform market is higher than that on B type of OS platform market.

\[
c_{\text{max}}^{A} > c_{\text{max}}^{B} 
\]

(3)

The application developers will develop applications on a highly leveraged OS platform based on the revenue sharing rates determined by the OS platform providers. Application developers’ selection to OS platforms is determined by type of application developer. The type of application developer is distributed according to the application development costs and it is given by \( \theta = c_{j}^{A} - c_{j}^{B} \). Then, The OS platform selection issue is decided by the type \( \theta \). Additionally, application developers will decide optimal application prices on the OS platform they have chosen.

Taking a brief look at this situation, there are a number of application developers who will develop their applications after choosing the OS platform on which they can maximize their profits. Application prices also affect the incomes of the application developers, as well as the incomes of the OS platform operators. A type of OS platform provider is supposed to determine the optimal rate of revenue sharing while the revenue sharing rate on B type of platform market is fixed. The revenue sharing rates determined by the OS platform providers have an impact not only on the incomes of OS platform providers but also application developers. Thus, each of two involved parties, OS platform providers and application developers, acts selfishly to maximize their profits respectively. Based on this, this situation where decision making affects each other can be game-theoretically analyzed.

### 2.2. Model Description

The situation presented above is a game situation between the A type of OS provider and N number of application developers, which are players of this game. At this point, the application developer’s set is defined by \( \mathbb{N} = \{1, 2, \cdots, n\} \). A strategy that A type of platform provider can select is a revenue sharing rate \( \alpha \) \((0 \leq \alpha \leq 1)\). A strategy \( j \)th application developers can choose is \( P_{A,j} \), the applications’ price \( j \)th application developers have determined.

Demand function of application can be expressed by utilizing platform markets’ latent demand on A type of OS platform \( S_{A} \), the price of application \( P_{A,j} \), and elasticity of demand \( b \). Given this, the application demand \( Q_{A,j} \) on A type of OS platform markets is defined by

\[
Q_{j}^{A} = S_{A} - bP_{j}^{A} 
\]

(4)

A type of OS provider’s profit \( \Pi_{A} \) is affected by the number of application developers who have chosen the OS provider, price of application and the rate of

revenue sharing that it has determined. Because there are two types of OS platforms that application developers can select, we define a set of application developers who choose A type of OS platform as follows: \( \mathbb{N}^A = \{ j \in \mathbb{N} | j - \text{th application developer chooses A - OS platform} \} \). The profit by selling applications on A type of OS platform is the sum of profit of applications sold by application developers who are in a set of \( \mathbb{N}^A \). A type of OS platform provider takes \( \alpha \), the percentage of revenue sharing rate, of the total revenue. As we assume that a marginal cost of the OS platform provider is zero for convenience, the profit of the OS platform is expressed as equation (5).

\[
\Pi_A(\alpha) = \sum_{j \in \mathbb{N}^A} \alpha Q^A_j p^A_j
\]  

(5)

The profit of jth application developer is determined by the price of application \( p^A_{j, A} \) which has been chosen on A type of OS platform and \( \alpha \) the OS platform provider has selected. Because the revenue of the application developer is \( Q^A_{A,j} p^A_{A,j} \) multiplied by \( (1 - \alpha) \), the profit of it should subtract the cost of application development \( C_{A,j} \) from the remaining value and then the profit can be expressed as follows:

\[
\pi_j(p^A_{A,j}) = (1 - \alpha)Q^A_{j, A} p^A_{A,j} - C_{j, A}
\]

(6)

The game model described above is shown in the following figure.
3. Analysis

Looking at the game situation, once A type of OS platform provider determines the revenue sharing rate, application developers are supposed to select one of two OS platforms based on the chosen revenue sharing rate. Then, the application developers decide the price of applications. In this situation, a Stackelberg game can be applied. Therefore, our analysis has used backward induction to analyze optimal revenue sharing rate of OS platform provider and optimal application price of application developers.

3.1 Optimal Application’s Price of Application Developer

Application developers know that application demand decreases as the application price increases and application demand increases as application price decreases. Thus, the application developers would like to set up an optimal application price to maximize their incomes. Since the income of the application developer is equal to the equation (6), the optimal application price derived from F.O.C (First Order Condition) is equal to the equation (7) as follows:

\[
\frac{\partial \pi_j(p^*_j)}{\partial p^*_j} = (1 - \alpha)(S_A - 2bP^*_j) = 0, \quad P^*_j = \frac{S_A}{2b}
\]  

(7)

The income, in which the optimal application price \(P^*_j\) is applied, of the OS platform provider \(\pi_j(P^*_j)\) is equal to the equation (8) as follows:

\[
\pi_j(P^*_j) = (1 - \alpha)\frac{S_A^2}{4b} - C^A_j
\]  

(8)

3.2 Application Developer’s OS Platform Choice Problem

In order to analyze the optimal revenue sharing rate of OS platform provider, application developer’s OS platform choice problem is considered in advance. In this study, we assume that there exist two types of OS platforms, namely, A and B. Thus, we can analyze the decision of the application developer’s market by comparing the application developers’ incomes with two types of OS platforms. Using the formula (8), the differences of application developer’s profit is derived, which is equivalent to the equation (9). Here, \(\beta\) is the fixed revenue share of the B type OS platform provider, \(S_B^2\) is latent demand of B type OS platform, and elasticity of demand for application price is assumed to be the same for both types of OS platforms.

\[
\pi_j(P^*_j) - \pi_j(P^*_B) = k - (C^A_j - C^B_j), \quad k = (1 - \alpha)\frac{S_A^2}{4b} - (1 - \beta)\frac{S_B^2}{4b}
\]  

(9)

Lemma 1: If both random variables \(C^A_j\) and \(C^B_j\) follow uniform distribution in \([0, C^A_{\text{max}}]\) and \([0, C^B_{\text{max}}]\) for each, and \(C^A_{\text{max}} > C^B_{\text{max}}\), then the probability that jth application developer choose A type OS platform, \(\text{Pr}(A)\) is as follows:
Proof: Application developer will release applications on a type of OS platform in case of \( \pi_j(P_{A,j}) - \pi_k(P_{B,j}) > 0 \). Accordingly, the choice problem of OS platform markets is determined by the value of \( k \) and to find out the probability that the application developer select a type of OS platform, a distribution of type \( \theta = C^A_j - C^B_j \) should be derived. Letting \( h(\theta) \) be the probability density function for \( \theta \), it can be expressed as follows by the convolution.

\[
    h(\theta) = \int_{-\infty}^{\infty} f_{C^A_j} f_{C^B_j}(\theta + C^B_j) dC^B_j
\]

where \( f_{C^A_j} \) is the joint distribution function of \( C^A_j \) and \( C^B_j \).

Since the random variables \( C^A_j \) and \( C^B_j \) are independent, equation (11) can be expressed as \( \int_{-\infty}^{\infty} f_{C^A_j}(\theta + C^B_j) f_{C^B_j}(C^B_j) dC^B_j \), and the random variable \( C^B_j \) follows a uniform distribution in \([0, C^B_{\max}]\), so \( h(\theta) \) is summarized as follows:

\[
    h(\theta) = \frac{1}{C^B_{\max}} \int_{0}^{C^B_{\max}} f(\theta + C^B_j) dC^B_j = \frac{1}{C^B_{\max}} \int_{-\infty}^{0} f(C^B_j) dC^B_j
\]

Equation (12) consists of three sections, and the probability density function for each section is as follows:

\[
    h(\theta) = \begin{cases} 
        \frac{C^B_{\max} + \theta}{C^B_{\max}C^A_{\max}} & \text{if } -C^A_{\max} < \theta < C^B_{\max} - C^A_{\max} \\
        \frac{1}{C^A_{\max}} & \text{if } C^B_{\max} - C^A_{\max} \leq \theta < 0 \\
        \frac{C^B_{\max} - \theta}{C^B_{\max}C^A_{\max}} & \text{if } 0 \leq \theta \leq C^B_{\max}
    \end{cases}
\]

Therefore, the application developer’s A-type OS platform selection probability according to \( h(\theta) \) is expressed by equation (14).

\[
    \Pr(A) = \Pr(\theta < k) = \int_{-\infty}^{k} h(\theta) d\theta
\]

Equation (14) consists of three sections according to the value of \( k \) by \( h(\theta) \). In case of \(-C^A_{\max} < k < C^B_{\max} - C^A_{\max}\),

\[
    \Pr(A) = \int_{-\infty}^{k} h(\theta) d\theta = \int_{-C^A_{\max}}^{C^A_{\max} + k} \frac{C^B_{\max}}{2C^A_{\max}C^B_{\max}} d\theta = \frac{(C^A_{\max} + k)^2}{2C^A_{\max}C^B_{\max}}
\]

In case of \( C^B_{\max} - C^A_{\max} \leq k < 0 \),
\[
\Pr(A) = \int_{-\infty}^{k} h(\theta)d\theta = \int_{-c_{A}^{\text{max}}}^{c_{B}^{\text{max}}-c_{A}^{\text{max}}} h(\theta)d\theta + \int_{c_{B}^{\text{max}}-c_{A}^{\text{max}}}^{k} h(\theta)d\theta = 1 - \frac{c_{B}^{\text{max}}-2k}{2c_{A}^{\text{max}}} \quad (16)
\]

In case of \( 0 \leq k \leq c_{B}^{\text{max}} \),
\[
\Pr(A) = \int_{-\infty}^{k} h(\theta)d\theta
\]
\[
= \int_{-c_{A}^{\text{max}}}^{c_{B}^{\text{max}}-c_{A}^{\text{max}}} h(\theta)d\theta + \int_{c_{B}^{\text{max}}-c_{A}^{\text{max}}}^{k} h(\theta)d\theta = 1 - \frac{(k-c_{B}^{\text{max}})^{2}}{2c_{A}^{\text{max}}c_{B}^{\text{max}}} \quad (17)
\]

<Q.E.D.>

### 3.3 Optimal Revenue Sharing Rate of OS Platform Provider

Profit of the OS platform provider is affected by revenue sharing rate and price application developer has chosen. While increasing rate of revenue sharing increases the profit of the OS platform provider, it decreases the profit of application developer. Since the application developer is less likely to choose the OS platform, the profit of the OS platform provider can consequently be lowered. Conversely, if the revenue sharing rate decreases, the profit of the OS platform provider also decreases. However, as the income of application developer increase, the probability that the application developer choose the OS platform also rises, which can ultimately make the income of the OS platform provider higher. As a result, it is important for the OS platform provider to derive the optimal revenue sharing rate by inducing application developers and maximizing its profit at the same time.

Assuming that the probability of choosing A type OS platform is the same for each developer, equation (5) representing the payoff of A type OS platform provider can be expressed as equation (18).

\[
\Pi_{A}(\alpha) = \sum_{j \in A} \alpha Q_{j} A^{j} P_{j}^{A} = \Pr(A) N \cdot Q_{j} A^{j} \quad (18)
\]

Proposition 1: Under the optimal application price, \( P_{j}^{A*} \), selected by the \( j \)th developer, the optimal revenue sharing rate of the OS platform provider is as follows:

\[
\alpha^{*} = \begin{cases} 
\frac{S_{A}^{2}-(1-\beta)S_{B}^{2}+4b\Delta_{A}^{\text{max}}}{3S_{A}^{2}} & \text{if} \quad 1 - \frac{(1-\beta)S_{B}^{2}+4b(C_{B}^{\text{max}}-C_{A}^{\text{max}})}{S_{A}^{2}} < \alpha < 1 - \frac{(1-\beta)S_{B}^{2}+4b\Delta_{A}^{\text{max}}}{S_{A}^{2}} \\
\frac{2b(2c_{A}^{\text{max}}-C_{B}^{\text{max}})+S_{A}^{2}-(1-\beta)S_{B}^{2}}{2S_{A}^{2}} & \text{if} \quad 1 - \frac{(1-\beta)S_{B}^{2}}{S_{A}^{2}} < \alpha \leq 1 - \frac{(1-\beta)S_{B}^{2}+4b(C_{B}^{\text{max}}-C_{A}^{\text{max}})}{S_{A}^{2}} \\
\frac{2\left(S_{A}^{2}-(1-\beta)S_{B}^{2}+4b\Delta_{B}^{\text{max}}\right)}{4b} & \text{if} \quad 1 - \frac{(1-\beta)S_{B}^{2}+4b\Delta_{B}^{\text{max}}}{S_{A}^{2}} \leq \alpha \leq 1 - \frac{(1-\beta)S_{B}^{2}}{S_{A}^{2}} \\
\end{cases} \quad (19)
\]

where \( D = \sqrt{\left(\frac{S_{A}^{2}-(1-\beta)S_{B}^{2}}{4b} - C_{B}^{\text{max}}\right)^{2} + 6C_{B}^{\text{max}}C_{B}^{\text{max}}} \)

Proof: In case of OS platform provider, it is necessary to determine \( \alpha \) to maximize the profit of the provider under the optimal application price determined by application developer. Under the optimal application price \( P_{j}^{A*} \), the payoff of the OS platform provider can be obtained by using equations (7) and (18), which is as follows:
\[ \Pi_A(\alpha) = \Pr(A) N \cdot \frac{S_A^2}{4b} \] (20)

Equation (20) is divided into three sections according to the value of \( k \). At this time, if each interval is expressed with respect to \( \alpha \), the profit of the OS platform provider can be expressed as follows:

In case of \(-C_{\text{max}}^A < k < C_{\text{max}}^B - C_{\text{max}}^A\), which also can be expressed as

\[
1 - \frac{(1-\beta)S_A^2 + 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} < \alpha < 1 - \frac{(1-\beta)S_A^2 - 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2},
\]

\[
\Pi_A(\alpha) = \frac{NS_A^2 \alpha}{8bc_{\text{max}}^B + c_{\text{max}}^A} \left( \frac{as_A^2}{4b} - \frac{s_A^2(1-\beta)S_A^2}{4b} - C_{\text{max}}^A \right)^2
\] (21)

Likewise, if

\[
1 - \frac{(1-\beta)S_A^2 + 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} < \alpha \leq 1 - \frac{(1-\beta)S_A^2}{S_A^2},
\]

\[
\Pi_A(\alpha) = \frac{NS_A^2}{8bc_{\text{max}}^B + c_{\text{max}}^A} \left[ -\frac{a^2S_A^2}{2b} + \alpha \left( \frac{s_A^2(1-\beta)S_A^2}{2b} + 2C_{\text{max}}^A - C_{\text{max}}^B \right) \right]
\] (22)

If

\[
1 - \frac{(1-\beta)S_A^2 + 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} \leq \alpha \leq 1 - \frac{(1-\beta)S_A^2}{S_A^2},
\]

\[
\Pi_A(\alpha) = \frac{NS_A^2}{8bc_{\text{max}}^B + c_{\text{max}}^A} \left[ -\alpha^2 \left( \frac{s_A^2}{4b} \right)^2 + \frac{a^2S_A^2}{2b} \left( \frac{s_A^2(1-\beta)S_A^2}{4b} - C_{\text{max}}^B \right) + 2\alpha C_{\text{max}}^A - C_{\text{max}}^B \right]^2
\] (23)

Since the profit of A type of OS platform provider by means of the revenue sharing rate is equal to equations (21) to (23), using F.O.C (first order condition), the optimal revenue sharing rate for each section can be derived.

In case of \( 1 - \frac{(1-\beta)S_A^2 + 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} < \alpha < 1 - \frac{(1-\beta)S_A^2 - 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} \), the optimal revenue sharing rate is equal to equations (24) and (25).

\[
\alpha^* = \frac{S_A^2 - (1-\beta)S_A^2 + 4bc_{\text{max}}^A}{3S_A^2}
\] (24)

\[
\alpha^* = \frac{S_A^2 - (1-\beta)S_A^2 + 4bc_{\text{max}}^A}{S_A^2}
\] (25)

Since the resultant \( \alpha^* \) derived from equation (25) is not included in

\[
1 - \frac{(1-\beta)S_A^2 + 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} < \alpha < 1 - \frac{(1-\beta)S_A^2 - 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2},
\]

only \( \alpha^* \) derived from equation (24) is the optimal revenue sharing rate. Given this, equation (23) has the condition of equation (25).

In case of \( 1 - \frac{(1-\beta)S_A^2}{S_A^2} \leq \alpha \leq 1 - \frac{(1-\beta)S_A^2 + 4b(c_{\text{max}}^B - c_{\text{max}}^A)}{S_A^2} \),

\[
\alpha^* = \frac{2b(2c_{\text{max}}^A - 4bc_{\text{max}}^B) + S_A^2(1-\beta)S_A^2}{2S_A^2}
\] (26)

such that \( 2b(3C_{\text{max}}^A - 2c_{\text{max}}^B) < S_A^2(1-\beta)S_A^2 \leq 2b(2c_{\text{max}}^A - C_{\text{max}}^B) \)

In case of \( 1 - \frac{(1-\beta)S_A^2 + 4bc_{\text{max}}^B}{S_A^2} \leq \alpha \leq 1 - \frac{(1-\beta)S_A^2}{S_A^2} \),
\[ \alpha^* = \frac{2\left(\frac{S_A^2 - (1-\beta)S_B^2 - C_{\text{max}}^B}{4b}\right)}{\frac{3S_A^2}{4b}} + K \] where \[ D = \sqrt{\left(\frac{S_A^2 - (1-\beta)S_B^2 - C_{\text{max}}^B}{4b}\right)^2 + 6C_{\text{max}}^A C_{\text{max}}^B} \] (27)

\[ \alpha^* = \frac{2\left(\frac{S_A^2 - (1-\beta)S_B^2 - C_{\text{max}}^B}{4b}\right)}{\frac{3S_A^2}{4b}} - K \] where \[ D = \sqrt{\left(\frac{S_A^2 - (1-\beta)S_B^2 - C_{\text{max}}^B}{4b}\right)^2 + 6C_{\text{max}}^A C_{\text{max}}^B} \] (28)

Here, \( \alpha^* \) derived from equation (28) is not included in

\[ 1 - \frac{(1-\beta)S_B^2 + 4bC_{\text{max}}^B}{S_A^2} \leq \alpha \leq 1 - \frac{(1-\beta)S_B^2}{S_A^2} \] only \( \alpha^* \) derived from equation (27) is the optimal revenue sharing rate. Given this, equation (27) has the condition of equation (29).

\[ 4bD - 8bC_{\text{max}}^B < S_A^2 - (1-\beta)S_B^2 < 4bD + 4bC_{\text{max}}^B \] (29)

\(<Q.E.D.>

4. Conclusion

In this work, we propose a Stackelberg game based revenue sharing situation between OS platform providers and application developers through game theory. In the situation, we analyze the optimal revenue sharing rate, developers’ OS platform choice problem, and the optimal price of applications. Importantly, we explore the revenue sharing problem and suggest a realistic model considering the optimal price and revenue sharing rate at the same time, which is different from other earlier studies.

However, we assume that a revenue sharing rate of an OS platform provider is fixed, so that competition between OS platform providers cannot be considered in this paper. Thus, it would be interesting to study the competitive model between OS platform providers, which may lead to a more realistic result.

Reference


